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AN AUTOMATIC ELECTRICAL ANALYZER FOR 45° STRAIN-ROSETTE DATA

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SUMMARY

A device that automatically analyzes the data from a 45° strain rosette is described. By means of an electrical circuit, containing as its principal elements five resistor-condenser combinations and three diode rectifiers, the rosette strains are properly combined to yield a direct meter reading of maximum shear strain and major and minor principal strains. Orientation of the major principal axis is indicated directly on the screen of a cathode-ray tube incorporated into the instrument. The analyzer may be adapted to indicate linear and shear strains in arbitrary directions about the test point and to compute stresses as well as strains. It is compact and can be easily assembled from standard electrical elements.

INTRODUCTION

Strain rosettes are now widely applied for the determination of magnitude and direction of the principal strains and magnitude of the maximum shear strain at selected points on a stressed surface.

The technique of the determinations consists in cementing three or more strain gages in close proximity on the member to be tested and observing the strain indications in the gages when the member is loaded. These data provide sufficient information for calculating the desired strain quantities either graphically or analytically (references 1 to 3). Klemperer, in reference 4, describes an electrical computer for analyzing data taken on three gages set at 0°-60°-120° apart or on four gages 0°-45°-90°-135° apart. Murray (reference 5) has suggested mechanical computers for analyzing data from rosettes consisting of gages arranged in the foregoing combinations, as well as from rosettes consisting of three gages 0°-45°-90° apart. In reference 6, Williams has described a method of producing a continuous oscillograph record of the Mohr circle of stress and strain at any point on the surface of a dynamically loaded member.

The present report describes a computer that yields, upon an input of the three measurements taken from a 0°-45°-90° rosette, an automatic indication of orientation of principal axes, major and minor principal strains, and maximum shear strain. An appendix is included to indicate how the instrument may be adapted to compute, in addition to the foregoing primary strain quantities, the linear and shear strains in any arbitrary direction about the test point. The instrument also suggests a method of obtaining an indication of magnitude and direction of principal stresses and strains directly from the rosette, without the intermediate process of observing the individual strain-gage readings.

Most strain-rosette data are first reduced to principal strains and their directions, and the principal strains are subsequently used to determine the principal stresses. The instrument herein described may also be readily adapted to indicate directly principal stresses and their directions, maximum shear stress, and stress at any desired angle; or the instrument may be built to simultaneously compute both strain values and stress values.

The author wishes to acknowledge the valuable contributions to the mechanical construction of the analyzer made by Messrs. Frank A. Friswold and Lewis C. Litzenberg of the Instrument Division at AERL.

The instrument was developed at the Aircraft Engine Research Laboratory of the NACA for use in analyzing data from strain rosettes attached to crankcases and other engine components. The experimental model was built in the spring of 1943.

SYMBOLS

The following symbols are used throughout the report:

ϵ_1, ϵ_2, ϵ_3 observed strain indications on gages 1, 2, and 3 of a 45° rosette, microinches per inch. Gage 1 is the reference gage and gages 2 and 3 are respectively orientated at 45° and 90° positive counterclockwise to gage 1.

ϵ_p, ϵ_q major and minor principal strains at point of surface under test, microinches per inch

γ_{max} maximum shear strain at point of surface under test, microinches per inch

θ_p angle of axis of major principal strain, degrees measured positive counterclockwise from reference gage 1 of rosette

DESCRIPTION

A view of the front panel of the analyzer is shown in figure 1(a); the electrical components are shown in figures 1(b) and 1(c).

Three knobs are provided for setting the input strains from the rosette. These knobs activate voltage taps on potentiometers placed across a 60-cycle input line. Rotating the knobs clockwise from the reference center position introduces voltages into the analyzer that represent positive strains; rotating them counterclockwise introduces voltages that represent negative strains. The input setting for each of the three knobs is indicated by a microammeter. A master selector switch is provided for denoting the quantity indicated by the meter. The input quantities, ϵ_1, ϵ_2, and ϵ_3, the maximum shear strain, the major principal strain, and the minor principal strain are indicated by the meter when the master selector switch is successively rotated to the positions indicating these quantities.

Three scales are marked on the meter: 0 to 100, 0 to 200, and 0 to 500; thus, most problems can be solved with the meter at a fair portion of its range. The same scale should be used, however, throughout a given analysis. Shunts for reducing the meter current by one-half and one-fifth are also provided. In some problems, the choice of a scale that permits the meter to be used at a fair portion of its range in the setting of ϵ_1, ϵ_2, and ϵ_3 results in a value of maximum shear strain or principal strain larger than the maximum value the meter can accommodate. When the meter is off scale, the shunting switches are introduced. The strain is then read on the scale that is used throughout the analysis, but the true strain is obtained by multiplying the meter reading by 2 or 5 depending on the scale factor used. A reversing switch for the meter is provided to indicate whether the principal strains are positive or negative.

Indication of the orientation of the major principal axis is accomplished by a cathode-ray oscillograph, the screen of which is covered with a calibrated radial-line scale. After the settings for the three strains from the rosette have been adjusted, a straight line appears on the oscillograph screen. This line passes through the origin; the portion on one side of the origin is longer than the portion on the other side. The shorter part is tapered at its end, but the longer part is of uniform width throughout its length with a bright spot at its extremity. The longer line points to the scale value that indicates the direction of the major principal axis referred to gage 1 of the rosette.

The principal components of the electrical circuit are the strain-setting potentiometers, five resistor-condenser combinations, and three diode rectifiers. The resistor-condenser combinations rotate the phase of the voltages between the taps of the potentiometers in such a way that the vector sums of combinations of these voltages directly yield expressions contained in the mathematical formulas for strain. The rectifiers permit scalar addition of out-of-phase voltages and produce direct current for deflecting the direct-current microammeter.

The cathode-ray tube is an adjunct to the electrical circuit. It acts as a ratiometer and indicates the ratio of two voltages, which, in turn, determines the orientation of the major principal axis.

OPERATION

The operation of the analyzer will be illustrated by the following example: Given the 45° strain-rosette data

Strain (microin./in.)		
ϵ_1	ϵ_2	ϵ_3
150	200	-250

In order to determine θ_p, γ_{max}, ϵ_p, and ϵ_q:

1. The data are examined to determine the most suitable scale, and it is seen that 0 to 500 is the lowest range that will include all the given strains. The master selector switch is brought to the ϵ_1 position, and the dial for ϵ_1 is turned clockwise until the meter reads 150. With the master selector switch in the ϵ_2 position, the ϵ_2 dial is turned clockwise until the meter reads 200. With the master switch in the ϵ_3 position the dial for ϵ_3 is turned counterclockwise (negative direction) until the meter reads 250.

2. The cathode-ray oscillograph is examined. The larger portion of the line through the origin points to a value of θ_p of 26°.

3. The master selector is turned to the γ_{max} position. Because the meter pointer is observed to be off scale, the meter scale factor is turned to 2. The meter reads 320; hence, $2 \times 320 = 640$ microinches per inch.

4. The master selector is turned to ϵ_p. The meter reads 270 when the meter factor is reset to unity; hence, $\epsilon_p = 270$ microinches per inch.

5. The master selector is turned to ϵ_q. The meter reads off scale to the left; ϵ_q must be negative. The meter polarity is switched to –. The meter reads 370; hence, $\epsilon_q = -370$ microinches per inch.

Many check examples have been solved with the analyzer, and the accuracy of the instrument for most examples was determined to be about 2 percent of full scale. The principal angle was correctly indicated to within 2°. The accuracy of the instrument is dependent upon the care exercised in choosing the circuit components. Several suggestions are made in a later section entitled Notes on Construction.

Whenever the maximum shear strain is very low, the line in the cathode-ray screen becomes short, and its direction is difficult to determine. The maximum shear strain can be close to zero, however, only when both $(\epsilon_2 - \epsilon_1)$ and $(\epsilon_2 - \epsilon_3)$ are very small; in other words, when all three strains are approximately equal. When this condition prevails, the strains in all directions about the test point become approximately equal and the orientation of principal axes loses its significance.

THEORY

The electrical circuit of the analyzer was designed to compute the individual quantities appearing in the formulas for strain and subsequently to combine these quantities to yield final strains and their orientation.

Maximum Shear Strain

By deduction from equations (1.30), (1.46), and (1.47) of reference 2

$$\gamma_{max} = \sqrt{2}\sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2} \qquad (1)$$

It is common knowledge in alternating-current theory that the vector sum of two sinusoidal voltages, 90° out of phase with each other, is numerically equal to the square root of the sum of the squares of the two voltages. The problem of electrically computing the shear strain is resolved to (1) producing two sinusoidal voltages proportional to $(\epsilon_1 - \epsilon_2)$ and $(\epsilon_2 - \epsilon_3)$; (2) rotating these voltages until they are at right angles to each other; and (3) adding the two voltages electrically.

The principle by which the operation is accomplished may be explained with the aid of figures 2 and 3. In figure 2, MN, RS, and TV are rheostat potentiometers connected across a 60-cycle alternating-current line. In operation, the taps A, C, and E are displaced from the electrical centers O of the potentiometers by distances proportional to ϵ_1, ϵ_2, and ϵ_3, respectively. The voltage between A and C is thus proportional to $(\epsilon_1 - \epsilon_2)$, and the voltage between C and E is proportional to $(\epsilon_2 - \epsilon_3)$. These two voltages are placed across impedances each consisting of a condenser and a matched resistor. The reactance of the condenser at the imposed frequency of 60 cycles is numerically equal to the resistance of the matched resistor. The vector diagrams for the resolution of the voltages are shown in figures 3(a) and 3(b). From figure 3(a) the voltage E_{BC} across the resistor BC is equal to $\frac{\sqrt{2}}{2} E_{AC}$ and leads E_{AC} by 45°; the voltage E_{CD} across the condenser CD is equal to $\frac{\sqrt{2}}{2} E_{CE}$ and lags E_{CE} by 45°. Now E_{AC} and E_{CE} are in phase since the resistance of the potentiometer is small in comparison with the impedance of the resistor-condenser combinations; hence, E_{BC} is at right angles to E_{CD}. If a voltmeter were placed between B and D it would register the vector sum of E_{BC} and E_{CD}, which, by figure 3(c), is proportional to $\frac{\sqrt{2}}{2} \sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2}$. Voltage E_{BE} therefore represents half the required shear strain. The factor 2 is taken into account in metering the voltage.

Major and Minor Principal Strains

The expression for the major principal strain (reference 2, equation (1.46)) is

$$\epsilon_p = \frac{\epsilon_1 + \epsilon_3}{2} + \frac{\sqrt{2}}{2} \sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2} \tag{2}$$

The square-root portion of equation (2) is represented by the voltage between B and D in figure 2; it is now necessary to obtain a voltage representing $(\epsilon_1 + \epsilon_3)$. In figure 4 the rheostat potentiometer WX has been added to those potentiometers already shown in figure 2. The tap G of the potentiometer WX is mechanically coupled to the tap E of the potentiometer TV, but the terminals of the two potentiometers are so connected across the input line that, as the knob for ϵ_3 is rotated, taps E and G move equal distances in opposite directions from the electrical centers O of the potentiometers. From figure 4 it is evident that the voltage between A and G is a measure of $(\epsilon_1 - \epsilon_3)$. In order to make point B one of the terminals across which the voltage representing $(\epsilon_1 + \epsilon_3)$ can be measured, the resistor-condenser combination CF-FG is included; the impedance between A and G then consists of two resistors BC and CF and two condensers AB and FG. The total capacitive reactance is chosen equal to the total resistance; hence, the voltage between B and F is equal to

$\frac{\sqrt{2}}{2} E_{AG}$, which is equal to $\frac{\sqrt{2}}{2}(\epsilon_1 + \epsilon_3)$. The factor $\sqrt{2}$, by which this quantity is greater than the desired $\frac{\epsilon_1 + \epsilon_3}{2}$, is again taken care of by adjusting the resistance of the circuit that measures this voltage.

The voltages representing the two expressions in equation (2) must now be added. Attention must be given to the fact that, although voltages E_{DB} and E_{FB} represent the desired strain combinations in equation (2), these voltages are nevertheless out of phase with each other. Provision must therefore be made for obtaining the scalar sum of the voltages, not the vector sum. The expedient used for eliminating the effect of the phase difference is the process of rectification. Figure 5 shows the measuring circuit; the lettered terminals of this circuit are connected to the correspondingly marked points of figure 4. The current due to each of the two voltages is independently rectified before being passed through the meter; hence, the current in the meter consists of two direct-current components, each proportional to one of the expressions in equation (2), plus numerous harmonics that do not deflect the direct-current meter. The direct-current components combine in scalar fashion to yield a net result proportional to the major principal strain. It is to be noted in figure 5 that the resistance of the meter must be small in comparison with R_3 and R_5 in order that no interaction exists between circuits DB and FB.

Consideration must be given to the algebraic sign of $(\epsilon_1 + \epsilon_3)$ in obtaining a measure of the principal strains. As shown in figure 5, the current in the meter due to E_{BF} is always upward regardless of whether $(\epsilon_1 + \epsilon_3)$ is positive or negative. A means must be provided for passing the current upward through the meter when $(\epsilon_1 + \epsilon_3)$ is algebraically positive and downward through the the meter when $(\epsilon_1 + \epsilon_3)$ is algebraically negative.

Figure 6 illustrates the principal used in the instrument for taking into account the algebraic sign of $(\epsilon_1 + \epsilon_3)$. From electrical considerations, the algebraic sign of $(\epsilon_1 + \epsilon_3)$ relates to whether the voltage between B and F is in phase or 180° out of phase with a reference voltage. Let E_o in figure 6 be this reference voltage. The reference voltage E_o' is arbitrarily introduced into the metering circuit BF as well as in a compensating circuit shunted across the meter. Consider first the case when the terminals B and F are shorted together; that is, when $(\epsilon_1 + \epsilon_3)$ is zero. Then the meter passes two equal and opposite currents due to E_o, and the net effect is a zero indication. Now consider the case when a voltage appears between B and F that is in phase with E_o'. The current flowing upward through the meter becomes greater than the original upward current, due to E_o' alone, by a value proportional to the voltage between B and F; the downward current is unaffected; hence, a net upward current proportional to $(\epsilon_1 + \epsilon_3)$ appears in the meter. If, on the other hand, the voltage between B and F is 180° out of phase with E_o', the net upward current is diminished by a value proportional to $(\epsilon_1 + \epsilon_3)$; the downward current is again unaffected and the net current flow in the meter is downward. Thus, the current in the meter due to $(\epsilon_1 + \epsilon_3)$ is either upward or downward, depending upon whether voltage E_{BF} is in phase or 180° out of phase with a reference voltage, or, equivalently, whether $(\epsilon_1 - \epsilon_3)$ is positive or negative. It is, of course, necessary that E_o' be greater than the maximum possible value of E_{BF} in order that the system be capable of supplying sufficient current in the downward direction.

The manner in which the auxiliary voltage E_o' is introduced into the measuring circuit of $(\epsilon_1 + \epsilon_3)$, by means of transformer T_3, is shown in figure 7. Inasmuch as E_{BF} is the voltage across the resistor of a resistor-condenser combination, the voltage E_o is likewise tapped from the resistor of a resistor-condenser combination in order that the two voltages will be in the proper phase.

The expression for the minor principal strain is identical with that given in equation (2) except that the sign preceding the square-root portion is negative. In order to obtain the minor principal strain, it is only necessary to reverse the rectifier of path DB in figure 5.

Direction of Major Principal Axis

From equation (1.48) of reference 2, the angle of the major principal axis, measured positive counterclockwise from the direction of gage 1, is given by

$$\tan 2\theta_p = \frac{2\epsilon_2 - (\epsilon_1 + \epsilon_3)}{(\epsilon_1 - \epsilon_3)} \tag{3}$$

This equation may be rewritten

$$\tan 2\theta_p = \frac{(\epsilon_2 - \epsilon_1) + (\epsilon_2 - \epsilon_3)}{(\epsilon_2 - \epsilon_3) - (\epsilon_2 - \epsilon_3)} \tag{4}$$

or

$$\frac{\tan 2\theta_p + 1}{\tan 2\theta_p - 1} = \frac{\epsilon_2 - \epsilon_3}{\epsilon_2 - \epsilon_1} \tag{5}$$

If, then, a means were devised for indicating the ratio of $(\epsilon_2 - \epsilon_3)$ to $(\epsilon_2 - \epsilon_1)$, this ratio would define a function of θ_p, which, in turn, would determine θ_p. A cathode-ray oscillograph is used in the instrument as a ratiometer. The circuit diagram of the oscillograph is shown in figure 8, but it is, of course, possible to use an external oscillograph of standard make.

The application of a cathode-ray oscillograph as a ratiometer is as follows: Two voltages representing $(\epsilon_2 - \epsilon_1)$ and $(\epsilon_2 - \epsilon_3)$ are present between CA and CE, respectively, (fig. 2). These two voltages are in phase and vary sinusoidally with time. If they are impressed across the horizontal and vertical deflection plates of a cathode-ray oscillograph, the horizontal and vertical displacements X_t and Y_t at any time t of the cathode beam are

$$X_t = K\,(\epsilon_2 - \epsilon_1) \sin 2\pi ft \tag{6}$$

$$Y_t = K\,(\epsilon_2 - \epsilon_3) \sin 2\pi ft \tag{7}$$

where K is the deflection sensitivities of both sets of plates, adjusted to equality, in inches per volt and f is the impressed frequency of 60 cycles per second.

If equation (7) is divided by equation (6)

$$\frac{Y_t}{X_t} = \frac{\epsilon_2 - \epsilon_3}{\epsilon_2 - \epsilon_1} \tag{8}$$

The path traced by the cathode-ray spot is thus a straight line through the origin whose slope is the desired ratio. The straight line, in conjunction with a radial scale, indicates θ_p. A convenient method of determining the radial scale is to assume consecutive values of θ_p and to determine from equation (5) the ratio $\frac{\epsilon_2 - \epsilon_3}{\epsilon_2 - \epsilon_1}$ that corresponds to each value of θ_p. The resulting scale is shown in figure 9.

It is seen from figure 9 that a straight line extending in both directions through the origin points to two angles, one at each end of the line. This ambiguity of angle is due to the fact that the value of the tangent of an angle does not in itself uniquely define the angle. The tangent must be considered as a fraction and the signs of the numerator and denominator must be individually examined in order to determine the quadrant in which the angle lies and thus the value of the angle itself. In the consideration at hand, the problem is to determine not only the polarity relation between $(\epsilon_2 - \epsilon_1)$ and $(\epsilon_2 - \epsilon_3)$ but also the absolute polarity of each of these quantities, taking the voltage input to the system as reference. By use of the input voltage to modulate the beam of the oscillograph, this polarity may be taken into account. The control-grid circuit of the cathode-ray tube contains a transformer driven from the input line (fig. 8). The transformer is chosen to bias the grid beyond cut-off at the negative peak of the input cycle, which removes the beam from the screen during a portion of the cycle. The undiminished line then points to a unique value of θ_p on the scale.

Stresses

It is possible to devise an analyzer to compute stress instead of strain. The formulas for maximum shear stress and principal stresses (equations (1.43), (1.46), (1.47), (1.53), and (1.54) of reference 2) contain the same strain combinations $(\epsilon_1 + \epsilon_3)$ and $\sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2}$ as those appearing in the formulas for strain, but the coefficients of these equations contain the modulus of elasticity and the Poisson's ratio of the material under test. In order to convert the strain analyzer to a stress computer, it is only necessary to change the values of the metering resistances and to add scales on the meter for stress. The same instrument may be made to indicate both stress and strain by the use of a toggle selector switch, or two meters may ve provided, one indicating strain and the other stress.

NOTES ON CONSTRUCTION

The construction of the analyzer may be facilitated and the accuracy of the instrument increased by following a definite procedure in choosing some of the circuit components.

1. Figure 7 shows five resistor-condenser combinations. It is important that each condenser be carefully chosen to have a capacitive reactance at 60 cycles, numerically equal to the resistance of its adjoining matched resistor. A convenient process for accomplishing a close match is to choose condensers of nominally correct capacitance (as specified in fig. 7) and to place each condenser, in turn, in series with a voltage source of 60 cycles and a variable resistor. The proper resistance for each condenser is determined by varying the resistance until the measured voltage across it is equal to the measured voltage across the condenser. Resistance R_7 should be determined in conjunction with the adjoining condenser C_2 when transformer T_3 is installed across R_7.

2. The operation of setting the strains ϵ_1, ϵ_2, and ϵ_3 into the instrument consists in moving the taps of the various potentiometers to points of potential that correspond to the strains. The reference potential is that of a fixed point in a resistor placed across the input line. Normally, this reference point would not be unique, and any point approximately halfway between the two input lines would be a satisfactory reference. Because the operation of the instrument depends, however, on the fact that, for any position of the dial for ϵ_3, the tap of one of the coupled potentiometers is as far above the reference point as the tap of the other potentiometer is below it, only one point in the reference resistor may be taken as the reference potential. In order to obtain this point, a sensitive voltmeter is first placed between the taps of the coupled potentiometers on the ϵ_3 shaft, and the ϵ_3 dial is rotated until the voltmeter reads zero. The voltmeter is then connected between one of the ϵ_3 taps and a tap on the reference resistor, which is adjusted for a zero reading of the voltmeter. The position of the tap on the reference resistor thus determined is the reference potential of the instrument. It is also important that the two coupled potentiometers representing ϵ_3 be accurately wound in order that, as the ϵ_3 knob is rotated, the two taps reverse equal resistances in opposite directions.

3. When a diode is placed in a relatively low-impedance circuit, a small current flows in the circuit even when no external voltage is applied. This phenomenon is due to the fact that emission takes place from the cathode and some electrons find their way to the

plate even when the plate is not positively charged. One expedient for reducing the current flow at an applied voltage of zero is to reduce the temperature of the cathode. For a 6H6 diode the normal filament voltage is 6.3 volts. It was found that approximately 3.5 volts to the filament provided adequate temperature to permit a full-scale current of 100 microamperes to flow. At the same time, the temperature was low enough to prevent undesirable emission.

4. The metering resistance R_3 is arbitrarily chosen to provide full-scale deflection of the meter for the full range of the ϵ_1, ϵ_2, and ϵ_3 dials. The other two metering resistances R_4 and R_5 are best determined by experiment. If, for example, ϵ_1 and ϵ_3 are set equal to zero and ϵ_2 is set to an arbitrary value, the meter reading, when the selector switch is set to the γ_{max} position, should be twice ϵ_2. The resistance R_4 is determined by adjusting its value until this condition prevails. Also, if ϵ_1, ϵ_2, and ϵ_3 are all set equal to an arbitrary value, the reading of the meter, when the master selector switch is in the ϵ_p position, should be equal to the arbitrary value of ϵ_1, ϵ_2, and ϵ_3. Resistance R_5 is adjusted until this condition prevails.

5. The accuracy of the instrument depends upon the constancy of the supply voltage throughout the analysis of a given set of rosette data. If the line voltage is likely to fluctuate, it is desirable to incorporate a voltage regulator in the instrument, but care should be exercised that no harmonics are introduced by the regulator.

6. Equations (6) and (7) are valid only if the deflection sensitivities of the vertical and horizontal plates are equal. Because of manufacturing tolerances in the cathode-ray tube, the sensitivities may not be identical; hence, resistor-gain controls are provided for adjusting them to equality.

DIRECT MEASUREMENT OF PRINCIPAL STRESSES AND STRAINS

The instrument suggests the development of a circuit for direct indication of magnitude and direction of principal stresses and strains from the strain-gage voltages without the intermediate process of observing ϵ_1, ϵ_2, and ϵ_3. In figure 4, for example, the purpose of the potentiometers is to provide a means of obtaining voltages proportional to $(\epsilon_1 - \epsilon_2)$, $(\epsilon_2 - \epsilon_3)$, and $(\epsilon_1 + \epsilon_3)$. A voltage proportional to the difference between the strain indications of two strain gages can be obtained from a Wheatstone-bridge arrangement in which the two strain gages constitute adjacent arms, and a voltage representing the sum of two strain-gage indications can be obtained if the gages are arranged as opposite arms of the bridge. A circuit similar in principle to that used in the analyzer, but with the potentiometers replaced by strain gages and fixed resistors, may thus make it possible to obtain direct indication of principal stresses and strains. Suitable amplifiers would be required in such a circuit for increasing the power level of the strain voltages.

A circuit for the direct indication of principal stresses and strains could possibly be designed with a simpler phase-rotating arrangement than that used in the analyzer for producing perpendicular voltages. One arrangement, originally considered for the analyzer, involved the use of a resonant resistor-condenser-inductor combination. It was found, however, that commercially available inductors of the required size varied appreciably with the current flow through them and that a resistor-condenser-inductor combination could not be readily devised to be resonant at all values of current flow. In a circuit for direct use with the strain gages, the currents in the phase-rotating elements would be considerably lower, and it is possible than an inductance would not be undesirable.

CONCLUDING REMARKS

It is recognized that the circuit constants, such as the voltages, the size of resistors and condensers, and the range of the meter used in the analyzer, could be combined to produce a cheaper or more compact instrument than the one described in this report. The present instrument represents an experimental model, made with materials at hand, to illustrate the manner in which the principles described may be combined to yield a satisfactory automatic analyzer for 45° strain-rosette data.

Aircraft Engine Research Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, April 12, 1944.

APPENDIX

LINEAR AND SHEAR STRAINS IN ARBITRARY DIRECTION

ABOUT THE TEST POINT

For some applications, it may be desired to know the linear or shear strains in any direction about the test point. Let ϵ_θ be the linear strain at an angle of θ degrees to the major principal direction. Then by equation (1.22c) of reference 2

$$\epsilon_\theta = \frac{\epsilon_p + \epsilon_q}{2} + \frac{\epsilon_p - \epsilon_q}{2} \cos 2\theta \tag{9}$$

From equation (1.33) of reference 2 it can be deduced that

$$\epsilon_p + \epsilon_q = \epsilon_1 + \epsilon_3 \tag{10}$$

and from equations (1.46) and (1.47) of the foregoing reference that

$$\epsilon_p - \epsilon_q = \sqrt{2}\sqrt{(\epsilon_2 - \epsilon_1)^2 + (\epsilon_2 - \epsilon_3)^2}$$

then equation (3) reduces to

$$\epsilon_\theta = \frac{\epsilon_1 + \epsilon_3}{2} \quad \frac{\sqrt{2}}{2} \sqrt{(\epsilon_2 - \epsilon_1)^2 + (\epsilon_2 - \epsilon_3)^2} \cos 2\theta \tag{11}$$

Equation (11) is the same as equation (2) except that an attenuating factor of $\cos 2\theta$ is applied to the square-root quantity. The strain at the arbitrary angle θ may then be determined by placing the master selector switch in the ϵ_p position and by attenuating the current due to the square-root quantity by the factor $\cos 2\theta$. This attenuation may be accomplished either by tapping off only a portion of the full voltage between B and D in figure 2, by increasing the resistance of the measuring circuit, or by shunting the rectifier with a resistance, which diminishes the effectiveness of the rectifier in producing direct current. Whatever method is used, provision must be made to reverse the rectifier for those values of θ in which $\cos 2\theta$ becomes negative.

The angle θ must be measured from the direction of the major principal axis but, because the major principal axis is indicated by the cathode-ray oscillograph, any direction at the test point can be related to the principal direction.

If provision for attenuation is made, it becomes possible to determine the orientation of the major principal axis without the aid of an oscillograph. The orientation is determined from the fact that the attenuator makes it possible to determine the strain in any direction relative to the principal axis and the strains in several directions are known from the primary data of the rosette. Thus, if the attenuator is arbitrarily moved through the range of θ, a position will be found for which the meter indication is ϵ_1 and the angle indicated by the attenuator is the angle of the direction of gage 1 with the direction of the major principal axis. Because the direction of gage 1 is known, the direction of the major principal axis can be determined. An ambiguity exists, however, as to whether the angle of the principal axis is to be measured clockwise or counterclockwise from gage 1. This ambiguity may be clarified by a check reading. The angular scale of the attenuator is again traversed until the meter reads the strain ϵ_2; the angle indicated is the angle between the major principal axis and gage 2, clockwise or counterclockwise. Four directions now exist, which are potential directions of ϵ_p. Two of these directions coincide, which establishes the exact direction of ϵ_p. Four rotatable prongs, which record the angles, are an aid in the determination.

The shear strain γ_θ at an angle θ with the major principal direction is (equation (1.29b) of reference 2).

$$\gamma_\theta = (\epsilon_p - \epsilon_q) \sin 2\theta = \sqrt{2} \sqrt{(\epsilon_2 - \epsilon_1)^2 + (\epsilon_2 - \epsilon_3)} \sin 2\theta \qquad (12)$$

Again an attenuation factor, this time $\sin 2\theta$, is necessary. The same attenuation device introduced for obtaining linear strain may be used, but the angular scale must be different to introduce the sine functions. The attenuation, however, must be applied when the master selector switch is in the γ_{max} position.

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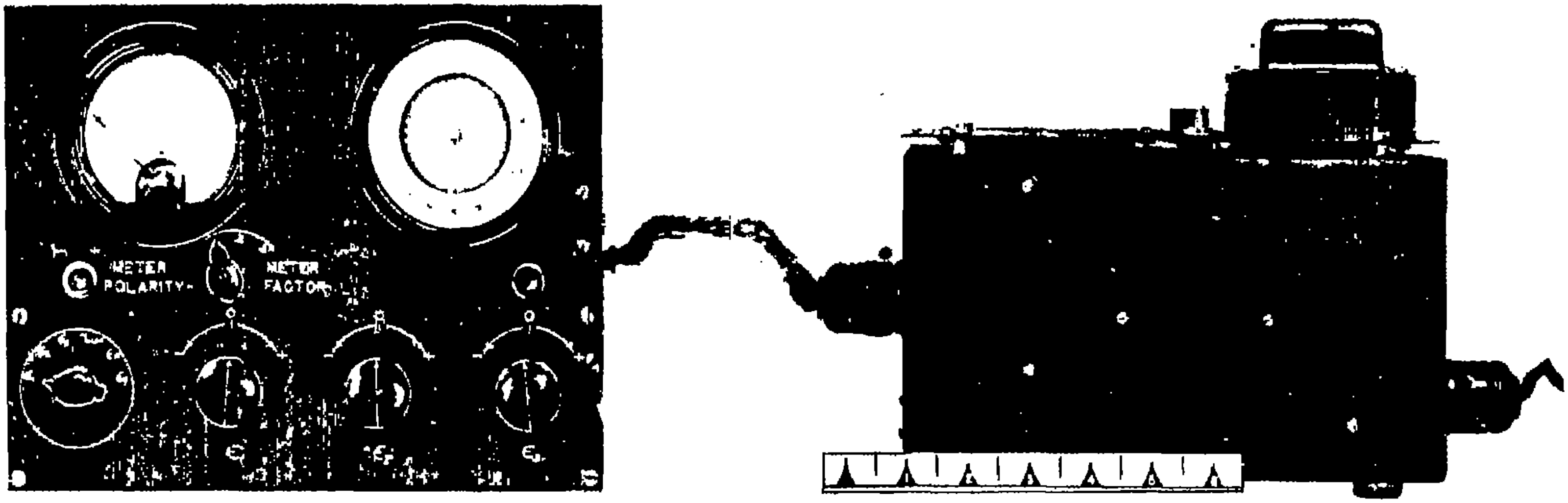
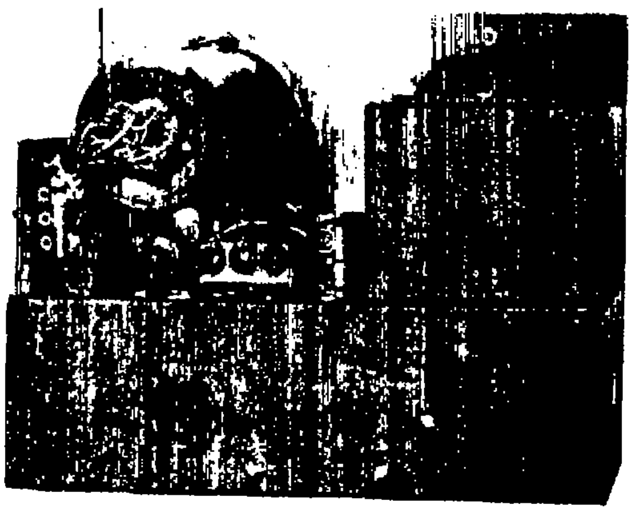
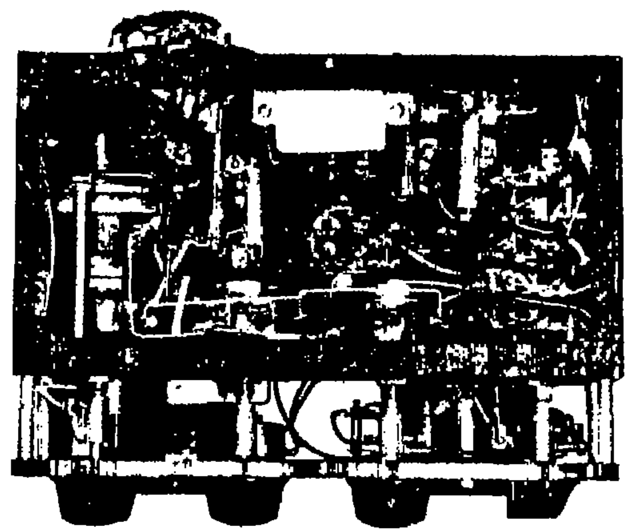


(a) Analyzer and power supply.



(b) Rear view.



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(c) Bottom view.

Figure 1. - Electrical analyzer for 45° rosette data.

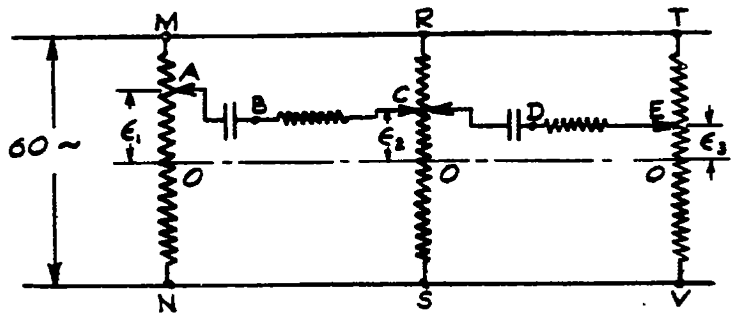
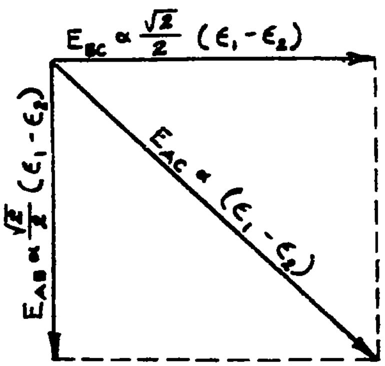
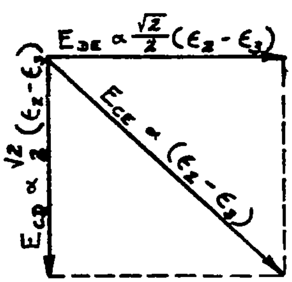


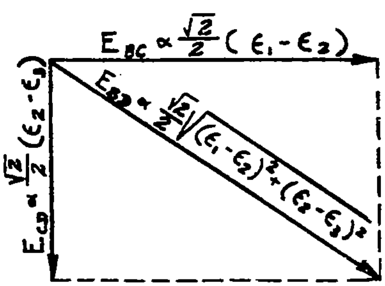
Figure 2. - Circuit showing manner of obtaining two perpendicular voltages proportional respectively to $(\epsilon_1 - \epsilon_2)$ and $(\epsilon_2 - \epsilon_3)$.



(a) Resolution of voltage E_{AC}.



(b) Resolution of voltage E_{CE}.



(c) Vector addition of E_{BC} and E_{CD}.

Figure 3. - Vector diagrams showing electrical determination of $\frac{\sqrt{2}}{2}\sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2}$ by addition of two perpendicular voltages.

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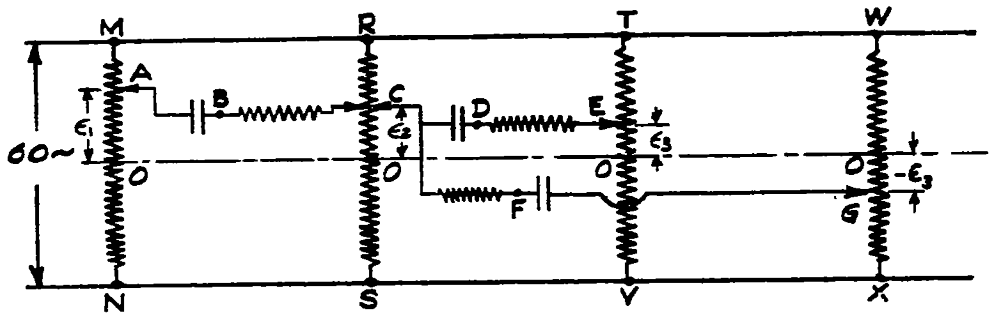

Figure 4. - Circuit showing manner of introducing voltage proportional to $(\epsilon_1 + \epsilon_3)$.

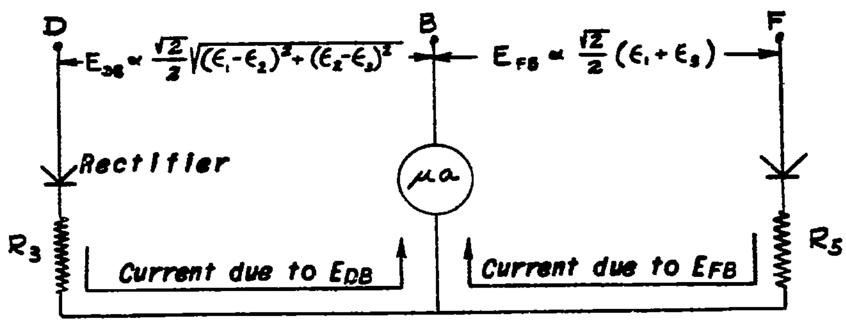

Figure 5. - Circuit for scalar addition of two out-of-phase voltages.

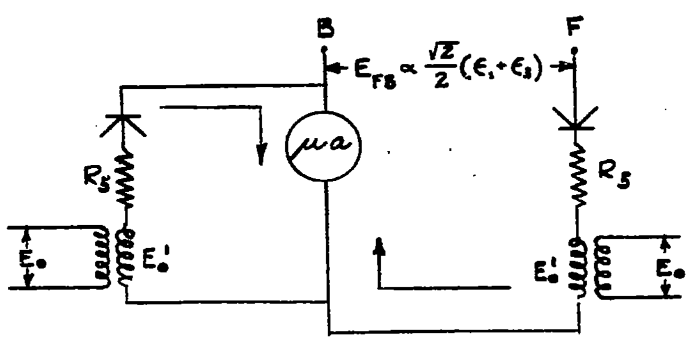

Figure 6. - Circuit showing method of providing for algebraic sign of $(\epsilon_1 + \epsilon_3)$.

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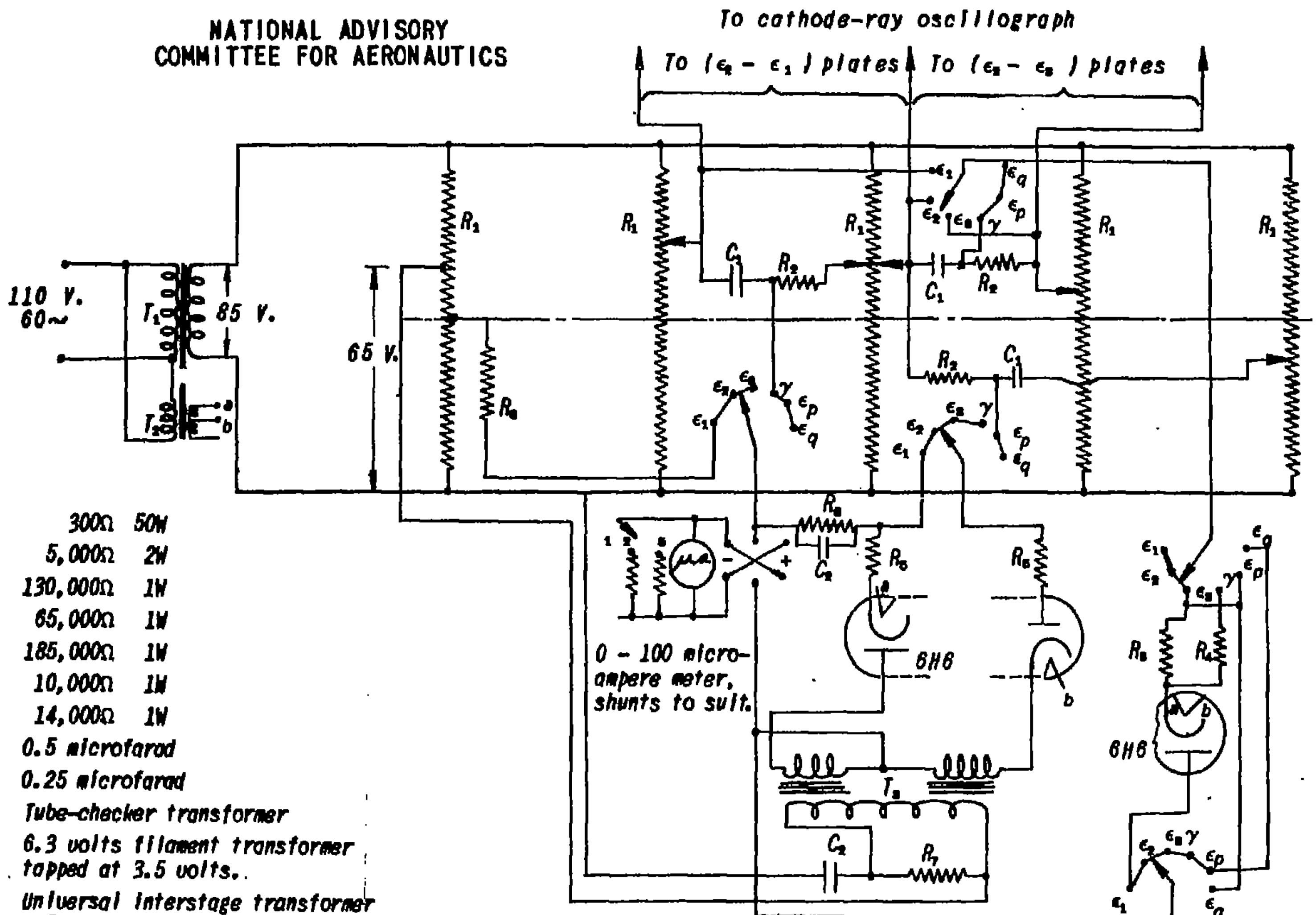
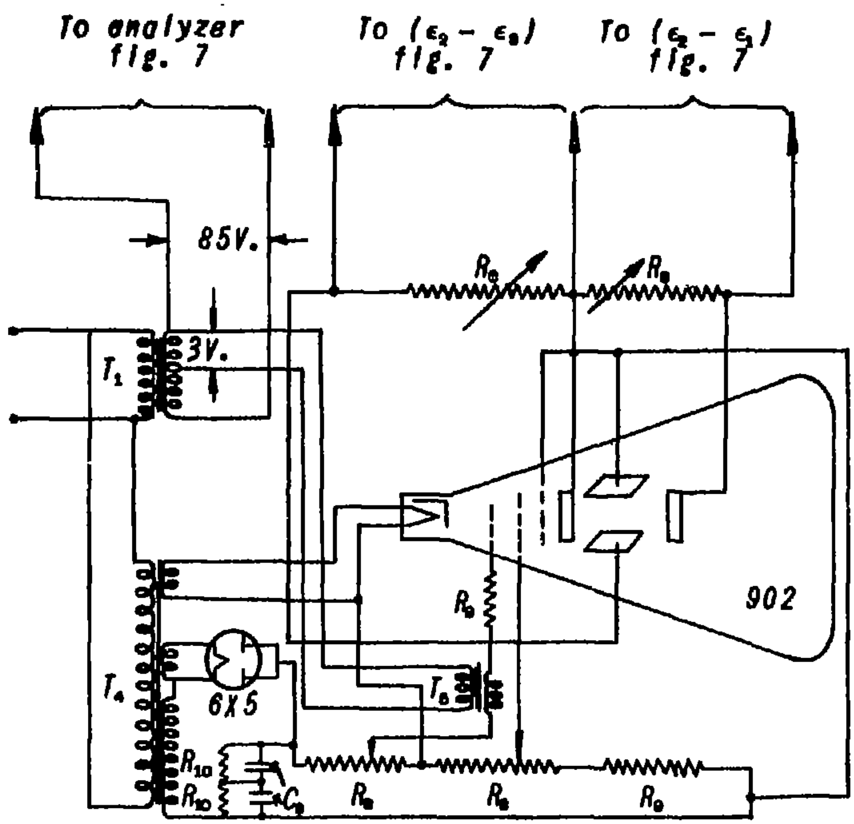


R_1	300Ω	50W
R_2	5,000Ω	2W
R_3	130,000Ω	1W
R_4	65,000Ω	1W
R_5	185,000Ω	1W
R_6	10,000Ω	1W
R_7	14,000Ω	1W
C_1	0.5 microfarad	
C_2	0.25 microfarad	
T_1	Tube-checker transformer	
T_2	6.3 volts filament transformer tapped at 3.5 volts.	
T_3	Universal interstage transformer 1.5 to 1 turns-ratio for each secondary winding.	

Figure 7. - Complete circuit diagram, excluding cathode-ray oscillograph, of electrical analyzer for 45° strain-rosette data.



R_8	100,000Ω 4W	
R_9	300,000Ω 1W	
R_{10}	75,000Ω 1W	
C_9	4.0 microfarads	
T_1	Tube-checker transformer	
T_4	Power transformer 6.3, 5.0, 650 volts secondaries	
T_5	Universal interstage transformer 1.5 to 1 turns-ratio secondary to primary.	

Figure 8. - Cathode-ray oscillograph for indication of orientation of major principal axis.

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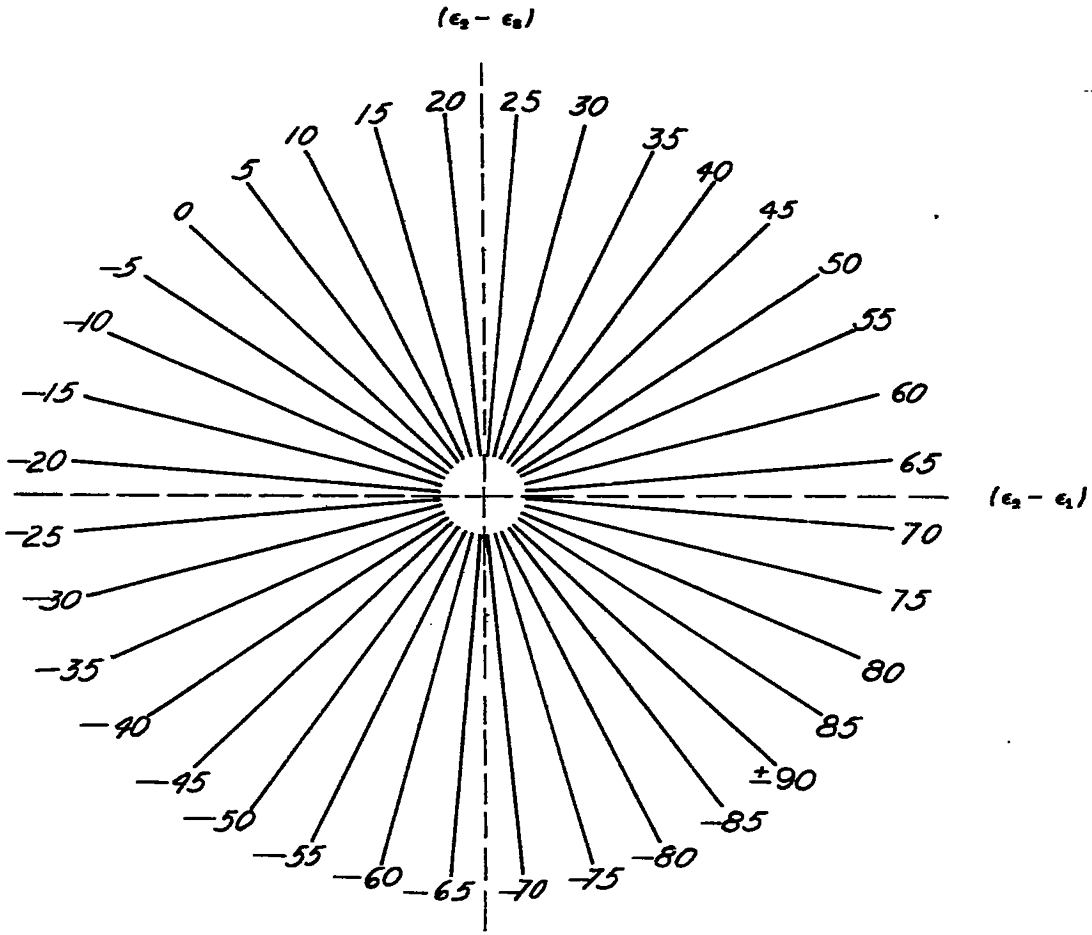

Figure 9. - Radial scale for determining θ_p when the plates of the cathode ray tube are in horizontal and vertical positions. Voltage $(\epsilon_2 - \epsilon_1)$ impressed on horizontal deflection plates, voltage $(\epsilon_2 - \epsilon_3)$ impressed on vertical deflection plates.

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